Chapter 3

The Coriolis force, geostrophy, Rossby waves and the westward intensification

The oceanic circulation is the result of a certain balance of forces. Geophysical Fluid Dynamics shows that a very good description of this balance is achieved if the oceans are subdivided into dynamical regions as sketched in Figure 3.1. We note that frictional forces are only important in the vicinity of ocean boundaries; in the vast expanse of the ocean interior below the surface layer they are negligible in comparison to the force set up by the pressure gradient. We know from Chapter 2 how to calculate the pressure field - subject to our choice of the depth of no motion - and should therefore be able to determine the flow in the largest of the dynamic regions.

The pressure gradient force cannot be the only acting force; otherwise the water would accelerate towards the centres of low pressure, as air movement on a non-rotating earth in Figure 1.1. Eventually, bottom friction would limit the growth of the velocity, and the circulation would become steady. Flow in the ocean interior is generally sluggish, and the friction force is no match for the pressure gradient force produced by variations in the density field. However, the pressure gradient force is not unopposed; it is balanced by a force which is known as the Coriolis force and is the result of the earth's rotation. It is an apparent force, that is, it is only apparent to an observer in a rotating frame of reference. To see this, consider a person standing on a merry-go-round, facing a ball thrown by a person from outside. To follow the ball the person would have to turn and therefore conclude that a force must be acting on the ball to deflect it from the shortest (straight) path. The person throwing the ball sees it follow a straight path and thus does not notice the force, and indeed the force does not exist for any person not on the merry-go-round.

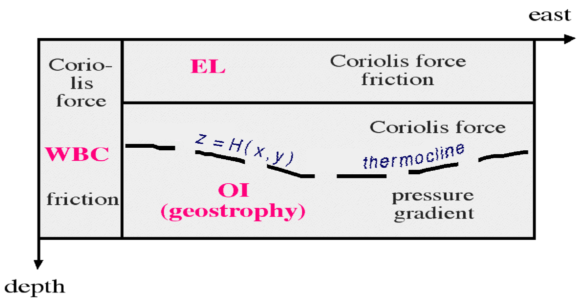

Fig. 3.1. An east-west section through an idealized ocean basin away from the equator showing the subdivision into three dynamic regions, the ocean interior (*OI*), the surface boundary or Ekman layer (*EL*), and the western boundary current region (*WBC*). In each region the Coriolis force is balanced by a different force. The $1^1/_2$ layer model discussed in the text divides the ocean interior further, into a dynamically active layer above the interface $z = H(x,y)$ and a layer with no motion below. The relative sizes of the various regions are not to scale.

In oceanography currents are always expressed relative to the ocean floor - which rotates with the earth - and can therefore only be described correctly if the Coriolis force is taken

Likewise, the transport per unit depth (1 m) between two stations *A* and *B* is given by

$$M' = \int_A^B \rho \, v_n \, dl \quad , \qquad (3.3)$$

where v_n now is the velocity normal to the line between *A* and *B*, and *M'* is the transport in the direction of v_n, with units of mass per unit depth and unit time (again kg m^{-1} s^{-1}). Unfortunately oceanographers refer to all three quantities (3.1), (3.2), and (3.3) as mass transport, and care has to be taken to verify which quantity is used in any particular study.

A quantity often found in oceanography is volume transport, defined as mass transport integrated over the width and depth of a current, divided by density. It has the unit m^3 s^{-1}. More commonly used is the unit Sverdrup (Sv), defined as 1 Sv = 10^6 m^3 s^{-1}.

The qualitative properties of geostrophic flow can be summarized as

Rule 1: In geostrophic flow, water moves along isobars, with the higher pressure on its left in the Southern Hemisphere and to its right in the Northern Hemisphere. In the ocean interior away from the equator, the flow of water is geostrophic.

The magnitude of geostrophic flow, expressed as mass transport per unit depth between two points *A* and *B*, is given to considerable accuracy over most of the ocean by:

$$M' = \frac{\rho_o \, g \, T_d \, \Delta h}{4\pi \sin \phi} = \frac{\rho_o \, g \, \Delta h}{f} \quad , \qquad (3.4)$$

where ρ_o is an average water density; g is the acceleration of gravity, $g = 9.8$ m s^{-2}; T_d is the length of a day = 86,400 s; ϕ is the latitude; and Δh is the difference in steric height between two adjacent steric height contours. $f = (T_d \, / \, 4\pi \sin\phi)^{-1}$ is known as the Coriolis parameter; it has the dimension of frequency and is positive north and negative south of the equator. Figure 3.2 is an illustration of Rule 1; it also demonstrates how the transport per unit depth between two streamlines can be evaluated.

Again, we can verify our rule by looking at the atmosphere (Figures 1.2 and 1.3). Whether the circulation is cyclonic or anticyclonic, high air pressure is always on the left of the wind direction in the Southern Hemisphere and to the right in the Northern Hemisphere. Meteorologists refer to this rule as Buys-Ballot's Law. We note at this stage that the equatorial region has to be considered separately, because the Coriolis parameter vanishes at the equator and another force is needed to balance the pressure gradient force.

Because *f* varies with latitude, the dependence of *M'* on *f* gives rise to waves of very large wavelength known as *Rossby waves*. To understand the mechanism of these waves it is useful to introduce an approximation to the ocean's density structure known as the "$1^1/_2$ layer ocean". In such a model the ocean is divided into a deep layer of contant density ρ_2 and a much shallower layer above it, again of constant density $\rho_1 = \rho_2 - \Delta\rho$. The lower

When the integral of (2.3a) is carried to some depth z_1 in the upper layer the result is

$$h(x, y, z_1) = \frac{(z_{nm} - z_1)(\rho_o - \rho_2) - (H(x, y) - z_1)\,\Delta\rho}{\rho_o} \quad . \qquad (3.5)$$

This does vary with position, because the interface depth $H(x,y)$ varies. The constant term $((z_{nm}-z_1)(\rho_2-\rho_o) - \Delta\rho\, z_1) / \rho_o$ does not affect horizontal differences and can be dropped; so the steric height is then just

$$h(x,y) = -\,\Delta\rho/\rho_o\, H(x, y) \; . \qquad (3.6)$$

Notice that horizontal gradients of steric height are independent of depth in the upper layer, so geostrophic flow in the upper layer will be independent of depth as well (see Figure 3.3).

The factor $\Delta\rho/\rho_o$ is of the order 0.01 or less; so $H(x,y)$ has to be much larger than $h(x,y)$. The negative sign in eqn (3.6) indicates that $H(x,y)$ slopes upward where $h(x,y)$ slopes downward and *vice versa*. At the surface, $h(x,y)$ measures the surface elevation needed to maintain constant weight of water, above every point on a depth level in the lower layer (see Figure 3.3). It follows that in a $1^1/_2$ layer ocean the sea surface is a scaled mirror image of the interface. This result is of sufficient importance to formulate it as

> **Rule 1a**: In most ocean regions (where the $1^1/_2$ layer model is a good approximation) the thermocline slopes opposite to the sea surface, and at an angle usually 100 - 300 times larger than the sea surface.

This is an important rule, because in contrast to the slope of the sea surface the slope of the thermocline can be seen in measurements made aboard a ship. This allows oceanographers to get a qualitative idea of currents from inspection of temperature and salinity data. If we now remember that Rule 1 links the direction of geostrophic flow with the sea surface slope we find that Rule 1a links the direction of geostrophic flow with the slope of the thermocline - a result we formulate as

> **Rule 2**: In a hydrographic section across a current, looking in the direction of flow, in most ocean regions (where the $1^1/_2$ layer model is a good approximation) the thermocline slopes upward to the right of the current in the southern hemisphere, downward to the right in the northern hemisphere.

Some consequences of Rule 1: Rossby waves and western boundary currents

We consider a $1^1/_2$-layer ocean - to define the sign of f, we take it in the southern hemisphere - with the bottom layer at rest. Suppose there is a large region in which the layer depth H is deeper than in surrounding regions, where both layers are at rest (*i.e.* H is constant there). Figure 3.4 shows a map of H for this situation. Appropriately scaled (by the factor $\Delta\rho/\rho_o$; see eqn (3.6) above) it is also a map of steric height, from which the flow at all depths in the upper layer can be deduced through eqn (3.4). It is seen that the feature represents a large anticyclonic eddy.

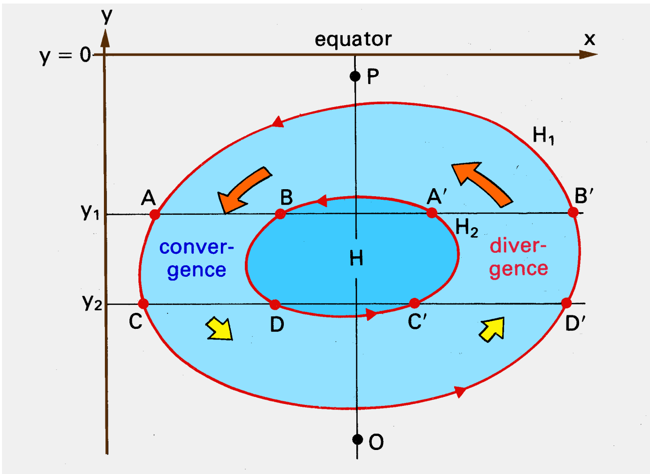

Fig. 3.4. Plan view of the eddy of Fig. 3.3. A and B are two points on the western side of the eddy at latitude y_1, on two isobars separated by an amount $\Delta h = \Delta\rho(H_1 - H_2))/\rho$ in steric height. C and D are two similar points at latitude $y_2 = y + \Delta y$. By eqn (3.4), total southward flow is greater in magnitude between A and B than between C and D because f is smaller in magnitude at A and B than at C and D; the thermocline deepens in $ABCD$. By the same argument, the thermocline shallows in $A'B'C'D'$: the eddy moves west.

Consider now the transport between two isobars corresponding to layer depths of H and $H + \Delta H$, at latitudes y_1 and y_2; ΔH is assumed to be small so the average depth of the layer is H. The total southward transport in the upper layer through the area between A and B is then $M_{tot} = H{\cdot}M'$ which, from eqns (3.4) and (3.6), is

$$M_{tot} = \frac{gH\,\Delta\rho\,\Delta H}{f(y_1)} = \frac{\rho_o\,gH\,\Delta h}{f(y_1)} , \qquad (3.7)$$

where $\Delta h = \Delta\rho\Delta H/\rho_o$ is the steric height difference between A and B, and $f(y_1)$ is the Coriolis parameter at the latitude y_1 of A and B. Similarly, at latitude y_2 between C and D:

The ratio $-(\partial H/\partial t) / (\partial H/\partial x)$ is the speed at which a line of constant H moves eastward. According to eqn (3.10) it has the constant value $-c_R(y)$ where

$$c_R(y) = \frac{\beta\, gH\, (\Delta\rho/\rho_o)}{f^2(y)} \qquad (3.11)$$

is called the Rossby wave speed. The sign of eqn (3.10) says that geostrophic eddies move *westward* with this speed. Notice that $c_R(y)$ approaches infinity rapidly at the equator, where $f = 0$.

Taking typical values of $H = 300$ m, $\Delta\rho/\rho_o = 3 \cdot 10^{-3}$, we find that $c_R(y)$ decreases from 1.27 m s^{-1} at 5°S or N to 0.08 m s^{-1} at 20°S or N and to 0.02 m s^{-1} at 40°S or N. At such speeds, a Rossby wave would take about 6 months to cross the Pacific at 5° distance from the equator, but more like 20 years at 40°.

Rossby waves are a general phenomenon in planetary motion of fluids and gases and occur in the atmospheres of the earth and other planets as well. In the earth's atmosphere they are usually better known as atmospheric highs and lows and play a key role in determining the weather. They generally move eastward, carried by the fast-flowing Westerlies. Relative to the mean flow of air, however, their movement is westward, as it has to be according to our discussion. Current velocities in the ocean are much smaller than the Rossby wave speed at least near the equator, so oceanic Rossby wave movement in the tropics and subtropics is towards west.

If the ocean were purely geostrophic - i.e. if Rule 1 applied *exactly* outside the western boundary currents - then the depressions and bulges in thermocline depth seen for example in the 500 m map of Figure 2.5 or in Figure 2.8 would all migrate to the western boundary through Rossby wave propagation, and the ocean would come to a state of horizontally uniform stratification and no flow. Thus, there must be some process continually acting to replenish these bulges. What is this process, and how does it work? This will be the subject of the next chapter.